ON THE MAGNETOELASTIC PROPERTY OF THE EARTH'S CRUST

by

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SUMMARY

The shifts of magnetic anomalies caused by earthquakes can be satisfactorily explained by the magnetoelasticity of the Earth's crust. Earthquakes bring about a rearrangement of elastic strain in parts of the crust, resulting in a lasting change of its average magnetic susceptibility. The estimative calculation carried out in the present paper shows that a change of susceptibility by some per cents is sufficient to modify the Earth's magnetic field by the actually observed amount. Moreover, an instantaneous strain rearrangement in the crust can — according to the calculations presented — generate an electromotive force of considerable intensity. However, the probability of observing the electromotive force deriving from the magnetoelastic effect is slight, as it will mostly be covered by telluric variations and by the seismoelectric phenomenon.

Introduction

As it is known, the susceptibility and remanent magnetic momentum of a ferromagnetic substance is strongly dependent on the mechanical strains and temperature [1]. Lately, the magnetostrictive behaviour of rocks was drawn into consideration in interpreting the results of palaeomagnetic investigations. It has been demonstrated that mechanical stresses will call forth important changes in the remanent magnetic momentum of rocks (2). Considering this all the assumption seems to be warranted that in the crustal zone of relatively great magnetic susceptibility, i. e. in the zone of temperature below the Curie-point, susceptibility will depend to a considerable degree on mechanical stresses. The Curie-point of rocks depends, according to investigations at the Carnegie Institute, but slightly on pressure. This means that, considering a Curie-temperature of 500 centigrades and a reciprocal geothermal gradient of 33 metres per centigrade, the ferromagnetism of rocks will prevail up to depths of 10 to 15 kilometres.

Instantaneous rearrangements of stress in the Earth's crust occur on the occasion of earthquakes.

The shift of magnetic anomalies attributed to an earthquake was first observed by H u m b o l d t. According to him the Cumana tremor of 1779 has brought about a change of 48′ in inclination. Similarly he measured a change of 28′ of inclination and a decrease by 5 per cent of the horizontal component in Lima, after an earthquake in 1802 (3).

Shifts of magnetic anomalies were observed by K a t o around Kuttuaro Lake in consequence of the earthquake of May 29, 1938 (4). With a similar point R o t h é has stated that the magnetic anomalies of the weakly seismic Paris basin are subject to temporal variation (5).

The above mentioned anomaly shifts are attributed to mass rearrangements and temperature and pressure changes. The latter will act upon the anomaly structure of the area in question by the means of influencing the magnetic susceptibility of the ferromagnetic rock complex. It is deemed to be unwarranted to assume mass rearrangements of great volume in the crust. Consequently, the observed changes have to be attributed to changes in magnetic susceptibility and remanent momentum. The range of possible strain variation is much broader than that of temperature variation. It is therefore assumed that the susceptibility and remanent magnetic momentum will be significantly modified by stress rearrangements in the first place.

In the present paper an attempt will be made to interpret local anomaly shifts solely by the hypothesis of "susceptibility change due to stress rearrangement". It will be further shown, that the instantaneous stress rearrangement occurring with earthquakes may generate a considerable electromotive force in the crust as well.

To estimate the intensity of the magnetoelastic effect to be expected we will construct a simple model. Let us assume that the mechanical stresses bring about a homogeneous isotropic change of magnetic susceptibility within a sphere of the radius R. The electrical conductivity of the sphere and its environment is taken to be zero. Let us study the local changes of horizontal and vertical intensity (δH and δZ) caused by a change in susceptibility in a horizontal plane above the sphere. The electromotive force will be studied by considering circular circuits in planes parallel to the above mentioned one. — It is not worth while to consider the sedimentary rocks of the crust. In magmatic rocks the susceptibility is of the order of magnitude of 10^{-3} to 10^{-4} cgs, much greater than in most sedimentary rocks (6).

Estimation of δH and δZ

The change in field intensity due to susceptibility change will be determined by the aid of the magnetic potential function.

The potential of the sphere of permeability μ situated in a medium of permeability μ' is given by the expression

$$^{k}\Phi = F\left(\frac{\mu - \mu'}{\mu + 2\,\mu'}\,\frac{R^3}{r^3} - 1\right)\xi$$

wherein F is the total intensity of the assumedly homogeneous terrestrial magnetic field, and the significance of ξ may be read from Fig. 1. The permeability μ' of the surrounding medium being constant and approximately equal unity, we have

$$^{k}\Phi = F\left(\frac{\mu - 1}{\mu + 2}\cdot\frac{R^3}{r^3} - 1\right)\xi\,.$$

Considering that

$$\mu = 1 + 4\,\pi\,\varkappa$$

($\varkappa$ being the magnetic susceptibility):

$$^k\Phi = F\left(\frac{4\,\pi\,\varkappa}{3 + 4\,\pi\varkappa}\cdot\frac{R^3}{r^3} - 1\right)\xi\,.$$

Hence, the change of magnetic potential due to a unit change of susceptibility is

$$^k\Phi'_\varkappa = \frac{3\cdot 4\,\pi\,F}{(3+4\,\pi\,\varkappa)^3}\cdot\frac{R^3}{r^3}\,\xi \approx \frac{4\,\pi\,F\,R^3}{3\,r^3}\,\xi\,;\quad (4\,\pi\,\varkappa \ll 1)\,.$$

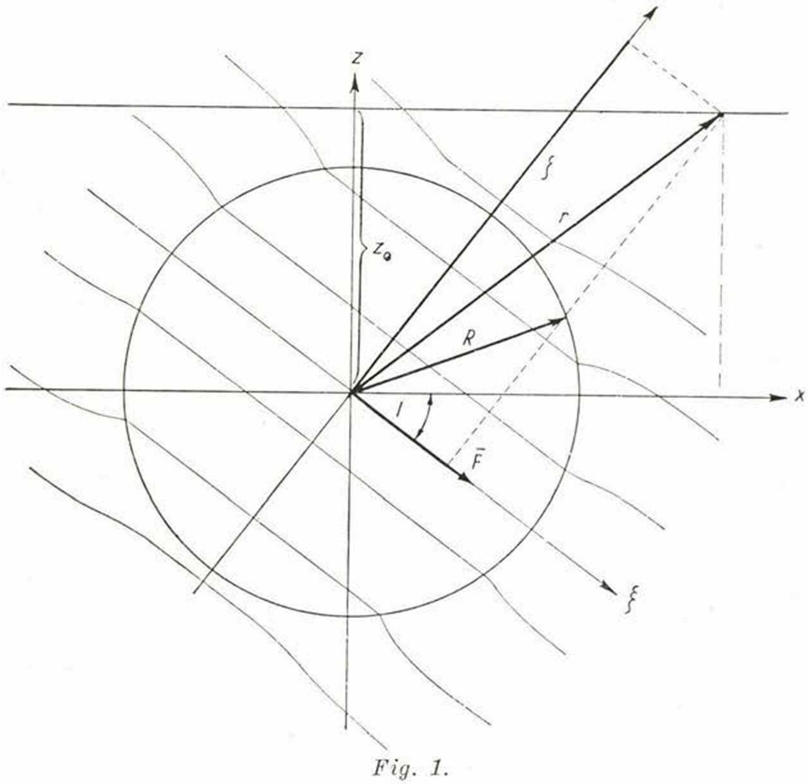

Fig. 1.

Applying the coördinate transformation

$$\xi = x\cos I - z\sin I$$

as shown by the figure, we obtain

$$^k\Phi'_\varkappa = \frac{4\,\pi\,F\,R^3}{3\,r^3}\,(x\cos I - z\sin I)\,.$$

The derivate of this expression in the direction ($-z$) yields the change of the z-component, $\delta\,Z$, corresponding to a unit change in susceptibility:

$$^kZ'_\varkappa = -\,\frac{\partial\,^k\Phi'_\varkappa}{\partial z} = \frac{4\,\pi\,F\,R^3}{3\,r^3}\left\{\sin I + \frac{3\,z\,(x\cos I - z\sin I)}{r^2}\right\}\,. \tag{1}$$

Hence

$$\delta Z = {}^kZ'_\varkappa \, \delta \varkappa.$$

A similar consideration yields the horizontal component of the field intensity change:

$$ {}^kH'_\varkappa = -\frac{\partial\, {}^k\Phi'_\varkappa}{\partial x} = \frac{4\pi F R^3}{3 r^3} \left\{ \frac{3x(x\cos I - z \sin I)}{r^2} - \cos I \right\} \qquad (2)$$

and

$$\delta H = {}^kH'_\varkappa \, \delta \varkappa.$$

Fig. 2 shows the course of the anomaly shift in the plane $z = z_0$, along the line (x, O, z_0), for $I = 63°$ and $\delta\varkappa = 1$.

Let us consider the absolute value of the bracket expressions in (1) and (2). We find that the maximum value of these expressions is of the order of magnitude of unity. Thus the order of magnitude of the anomaly will be determined by the coefficient

$$\frac{4\pi F R^3}{3 r^3} \delta \varkappa.$$

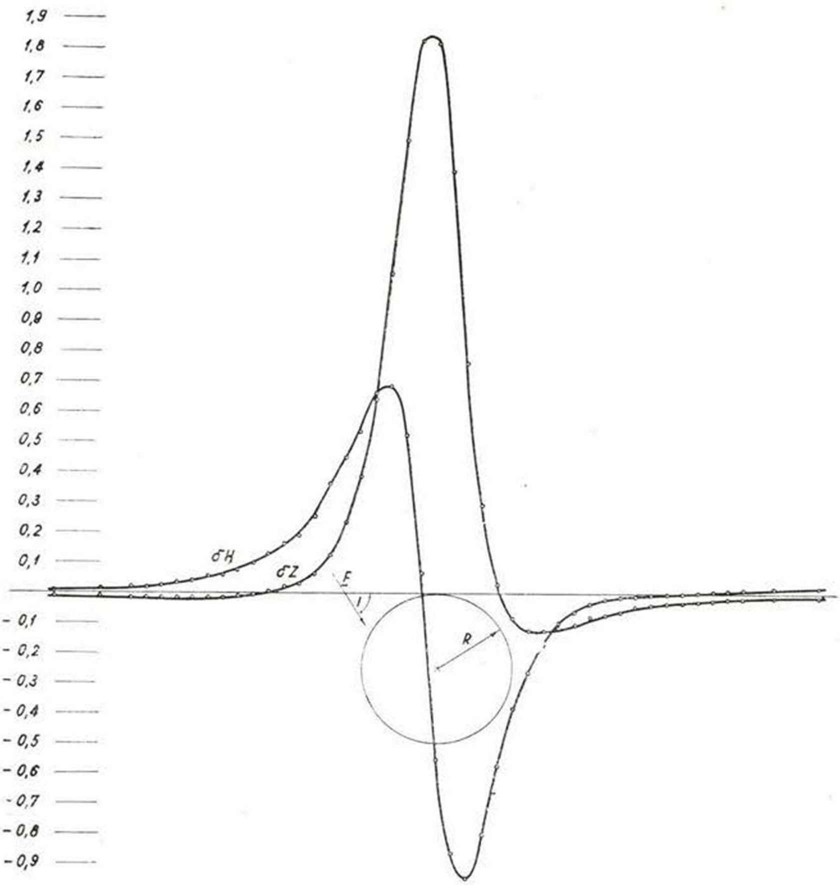

Fig. 2.

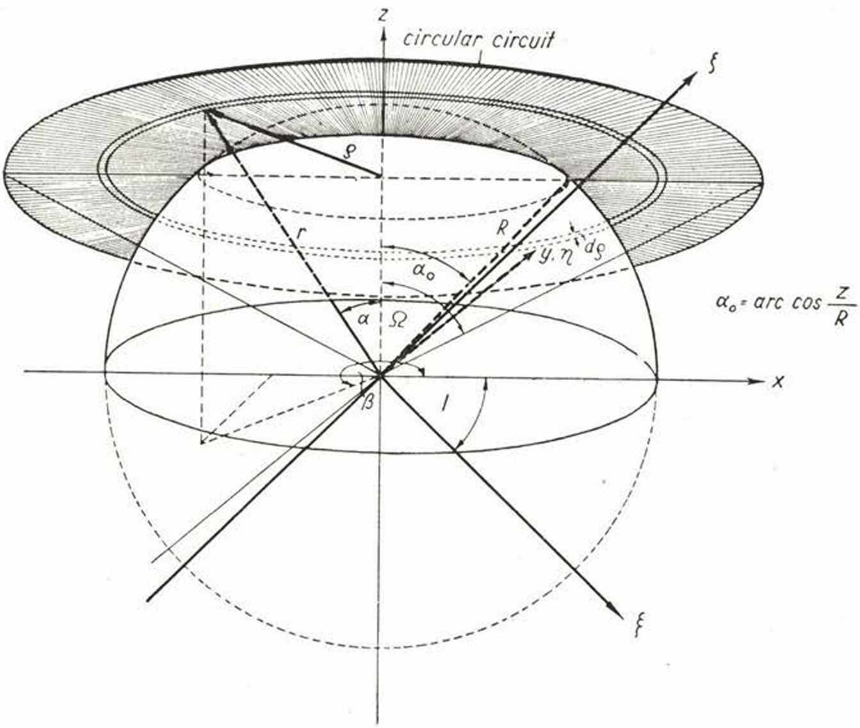

Fig. 3.

Let us assume a change of $\varkappa$ by 5 per cent due to stress rearrangement within the sphere R, i. e. $\delta\varkappa$ to be 0,05.

Then

$$\frac{R^3}{r^3}\gamma \leqslant \frac{4\,\pi\,F\,R^3}{3\,r^3}\,\delta\,\varkappa \leqslant 10\,\frac{R^3}{r^3}\,\gamma\,.$$

Consequently, in case of a favorable R/r ratio the resulting shift may reach the order of magnitude of 10γ ($R \leqslant r$). There will be points in which the values of anomaly change will agree well with the actually observed ones.

On the electromotive force generated by earthquakes

The change of susceptibility caused by the stress rearrangement due to an earthquake will engender a change in the magnetic flow and consequently generate an electromotive force in the crust. Let us consider the electrical circuit potential along a circle of the center $(0,0,\,z)$, whose plane is perpendicular to the Z axis and whose angle of vision from the center of the sphere is $2\,\Omega$. The specific change of flow φ'_x, surrounded by the circuit, will generally be composed of two parts : of ${}^{k}\varphi'_{\varkappa}$, the flow of the circular ring situated without the sphere R and of ${}^{b}\varphi'_{\varkappa}$, the change of flow surrounded by the circle of intersection of the sphere and the horizontal plane (Fig. 3).

$$\varphi'_{\varkappa} = {}^{k}\varphi'_{\varkappa} + {}^{b}\varphi'_{\varkappa}\,.$$

The value of ${}^k\varphi'_\varkappa$ is given by the integral

$$\underbrace{\frac{\mathrm{d}}{\mathrm{d}\,\varkappa}\int B_n\,\mathrm{d}\,f}_{\text{circular ring}} \approx \underbrace{\int {}^kZ'_\varkappa\,\mathrm{d}\,f}_{\text{circular ring}}.$$

To compute the integral we introduce polar coördinates:

$$x = r\sin\alpha\cos\beta\,; \qquad z = r\cos\alpha$$

$${}^kZ'_\varkappa = \frac{4\,\pi\,F\,R^3}{3\,z^3}\cos^3\alpha\,\{\sin I + 3\cos\alpha\,(\sin\alpha\cos\beta\cos I - \cos\alpha\sin I)\}.$$

The surface element df will be expressed as

$$df = \varrho\,\mathrm{d}\beta\,\mathrm{d}\varrho = \frac{z^2\sin\alpha}{\cos^3\alpha}\,\mathrm{d}\alpha\,\mathrm{d}\beta.$$

Consequently,

$${}^k\varphi'_\varkappa = \frac{4\,\pi\,F\,R^3}{3\,z}\int\limits_{\alpha=\alpha_0}^{\alpha=\Omega}\int\limits_0^{2\pi}\sin\alpha\,\{\sin I + 3\cos\alpha\,(\sin\alpha\cos\beta\cos I - \cos\alpha\sin I)\}\mathrm{d}\beta\,\mathrm{d}\alpha$$

$$\alpha_0 = \begin{cases}\arccos\dfrac{z}{R} & \text{if } 0 \leqq \dfrac{z}{R} \leqq 1.\\[2ex] 0 & \text{if } \dfrac{z}{R} > 1.\end{cases}$$

The term $\arccos z/R$ is the half angle of vision of the circle of intersection as regarded from the center of the sphere.

The above integral yields the value

$${}^k\varphi'_\varkappa = \begin{cases}\dfrac{8\,\pi^2\,R^3\sin I}{3\,z}\left[\cos^3\Omega - \cos\Omega + \dfrac{Z}{R} - \dfrac{Z^3}{R^3}\right]F & 0 \leqq \dfrac{z}{R} \leqq 1\,.\\[2ex] \dfrac{8\,\pi^2\,R^3\sin I}{3\,z}\,[\cos^3\Omega - \cos\Omega]\,F & \dfrac{z}{R} > 1\,.\end{cases}$$

To obtain ${}^b\varphi'_\varkappa$ we take the potential of the magnetic field within the sphere:

$${}^b\Phi = -\frac{3\,F}{3 + 4\,\pi\,\varkappa}\,.$$

By a procedure analogous to that of determining ${}^kZ'_\varkappa$ we get

$${}^b\Phi'_\varkappa \approx \frac{4\,\pi\,F}{3}\,(x\cos I - z\sin I)\,.$$

Hence

$${}^bZ'_\varkappa = -\frac{\partial\,{}^b\Phi'_\varkappa}{\partial z} = \frac{4\,\pi\,F\sin I}{3}\,.$$

The field being homogeneous, ${}^{b}\varphi'_{\varkappa}$ is obtained by multiplying the surface by ${}^{b}Z'_{\varkappa}$. The surface to be considered is, according to the figure

$$f = (R^2 - z^2)\,\pi .$$

Consequently

$${}^{b}\varphi'_{\varkappa} = \frac{4\,\pi^2\,F}{3} \sin I \cdot (R^2 - z^2)$$

and

$$\varphi'_{\varkappa} = \begin{cases} \dfrac{4\,\pi^2\,F}{3} \sin I \left[R^2 - z^2 + \dfrac{2\,R^3}{z} \left(\cos^3 \Omega - \cos \Omega + \dfrac{z}{R} - \dfrac{z^3}{R^3} \right) \right] & 0 \leqslant \dfrac{z}{R} \leqslant 1 \\ \dfrac{8\,\pi^2\,F}{3} \sin I \dfrac{R^3}{z} \left[\cos^3 \Omega - \cos \Omega \right] & \dfrac{z}{R} > 1 \end{cases}$$

where

$$\operatorname{arc\,cos} \frac{z}{R} \leqslant \Omega \leqslant \frac{\pi}{2} .$$

The value of $\varphi'_{\varkappa}$ at $z = 0$ may be obtained by taking into consideration the restriction made for Ω by the aid of a limit computation.

$$(\varphi'_{\varkappa})_{z=0} = \lim_{z \to 0} \frac{4\,\pi^2\,F}{3} \sin I \left[R^2 - z^2 + \frac{2\,R^3}{z} \left(\cos^3 \Omega - \cos \Omega + \frac{z}{R} - \frac{z^3}{R^3} \right) \right] =$$

$$= \frac{4\,\pi^2\,F}{3} \sin I \left[R^2 - z^2 + \frac{2\,R^3}{z} \left(\frac{z^3}{r^3} - \frac{z}{r} + \frac{z}{R} - \frac{z^3}{R^3} \right) \right]_{z=0} =$$

$$= \frac{4\,\pi^2\,F}{3} \sin I \left[R^2 - 2\,R^3 \left(\frac{1}{R} - \frac{1}{r} \right) \right] . \qquad (3)$$

The voltage induced in the circuit of the angle of vision of $2\,\Omega$ by a susceptibility change of $\delta\,\varkappa$ during the time interval dt will be

$$V = \begin{cases} -10^{-8} \dfrac{\delta\,\varkappa}{\mathrm{d}\,t} \dfrac{4\,\pi^2\,F}{3} \sin I \left[R^2 - z^2 + \dfrac{2\,R^3}{z} \left(\cos^3 \Omega - \cos \Omega + \dfrac{z}{R} - \dfrac{z^3}{R^3} \right) \right] & \\ & 0 \leqq \dfrac{z}{R} \leqq 1, \\ -10^{-8} \dfrac{\delta\,\varkappa}{\mathrm{d}\,t} \dfrac{8\,\pi^2\,F}{3} \sin I \dfrac{R^3}{z} (\cos^3 \Omega - \cos \Omega) & \dfrac{z}{R} > 1, \end{cases}$$

considering the relation

$$V = -\,10^{-8} \frac{\delta\,\varkappa}{\mathrm{d}\,t} = -\,10^{-8} \frac{\mathrm{d}\,\varphi}{\delta\,\varkappa} \frac{\delta\,\varkappa}{\mathrm{d}\,t} = -\,10^{-8}\,\varphi'_{\varkappa} \frac{\delta\,\varkappa}{\mathrm{d}\,t} .$$

The value of the circuit potential is maximum for the circle of infinite radius lying in the plane $z = 0$:

$$V_{\max} = -\,10^{-8}\,4\,\pi^2\,F \sin I\, R^2 \frac{\delta\,\varkappa}{\mathrm{d}\,t} .$$

This follows from the expression

$$V = -10^{-8}\frac{4\pi^2 F}{3}\sin I\left[R^2 + 2R^3\left(\frac{1}{R}-\frac{1}{r}\right)\right]\frac{\delta\varkappa}{\mathrm{d}t}\,.$$

One third of this voltage arises along the horizontal great circle of the square :

$$V = -\frac{10^{-8}\,4\pi^2 F}{3}\sin I\; R^2\frac{\delta\varkappa}{\mathrm{d}t}\,.$$

The variation of the circuit potential $V = \oint E_s\,\mathrm{d}s$ with the distance z in different horizontal planes is seen in Fig. 4. For Curve I, $\varrho = R$, for Curve II, $\varrho = 2R$, ϱ being the radius of the circular loop. As to the order of magnitude of the electromotive force arising, the value $10^{-3}R^2 - 10^{-2}R^2$ Volts is obtained for the plane $z = 0$, with $\delta\varkappa = 0{,}05$, $\mathrm{d}t = 1$, $I = 63°$ (R in kilometres).

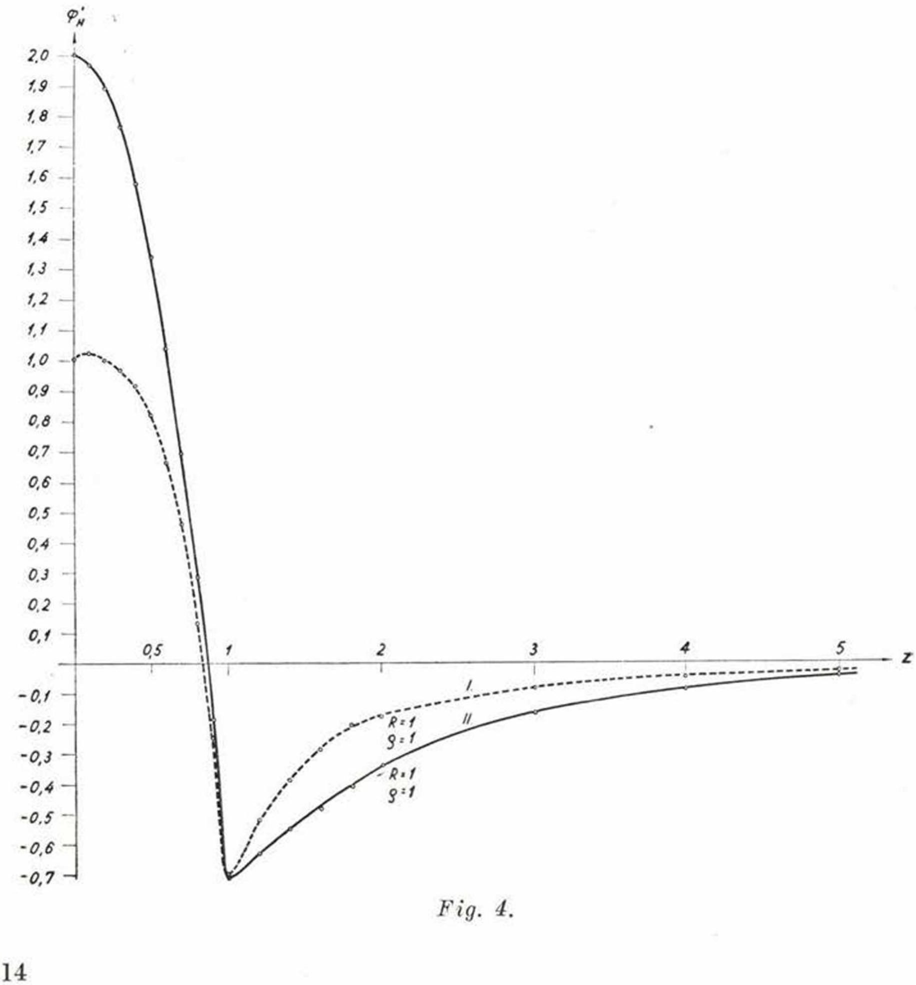

Fig. 4.

Conclusions

If considering the magnetoelastic effect, the magnetic anomaly shifts occurring in connection with earthquakes may be satisfactorily interpreted. The assumption of a change of susceptibility by some per cents is a much more probable one than the assumption of mass rearrangements of temperature changes.

The observation of the arising electromotive force on the surface will in most probability be seriously handicapped by telluric variations and by the seismoelectric phenomenon. However, an eventual observation of this effect will possibly shed useful light upon the mechanics of earthquakes.

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